Finite state transducers

Data Structures and Algorithms for Computational Linguistics III
(ISCL-BA-07)

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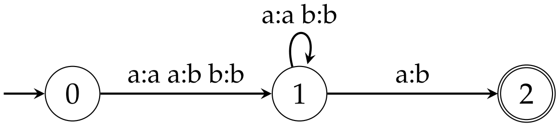
Finite state transducers

A quick introduction

- A *finite state transducer* (FST) is a finite state machine where transitions are conditioned on pairs of symbols
- The machine moves between the states based on an *input* symbol, while it outputs the corresponding *output* symbol
- An FST encodes a *relation*, a mapping from a set to another
- The relation defined by an FST is called a *regular* (or *rational*) relation



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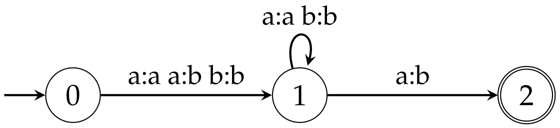


`babba` $\longrightarrow$ `babbb`

`aba` $\longrightarrow$ `bbb`

`aba` $\longrightarrow$ `abb`

Formal definition

A finite state transducer is a tuple $(\Sigma_i, \Sigma_o, Q, q_0, F, \Delta)$

Σ_i	is the *input* alphabet
Σ_o	is the *output* alphabet
Q	a finite set of states
q_0	is the start state, $q_0 \in Q$
F	is the set of accepting states, $F \subseteq Q$
Δ	is a relation $(\Delta : Q \times \Sigma_i \to Q \times \Sigma_o)$

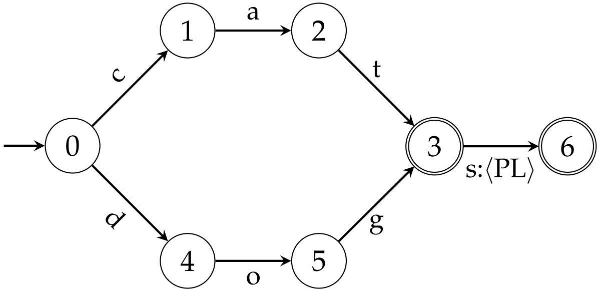
Where do we use FSTs?

Uses in NLP/CL

- Morphological analysis
- Spelling correction
- Transliteration
- Speech recognition
- Grapheme-to-phoneme mapping
- Normalization
- Tokenization
- POS tagging (not typical, but done)
- partial parsing / chunking
- ...

Where do we use FSTs?

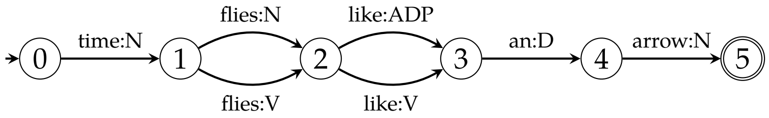
example 1: morphological analysis

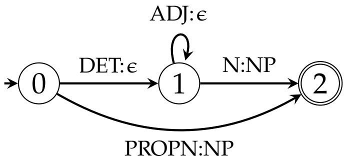


In this lecture, we treat an FSA as a simple FST that outputs its input:
the edge label ‘a’ is a shorthand for ‘a:a’.

Where do we use FSTs?

example 2: POS tagging / shallow parsing





Note: (1) It is important to express the ambiguity. (2) This gets interesting if we can 'compose' these automata.

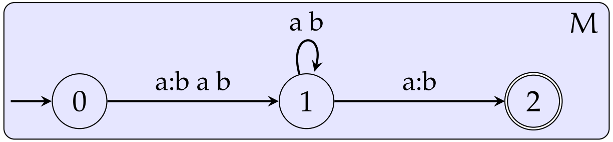
Closure properties of FSTs

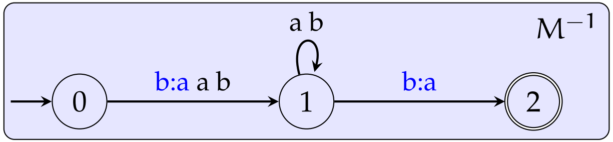
Like FSA, FSTs are closed under some operations.

- Concatenation
- Kleene star
- ~~Complement~~
- Reversal
- Union
- ~~Intersection~~
- *Inversion*
- *Composition*

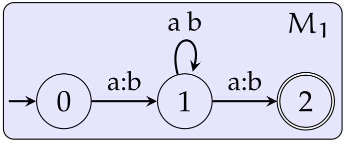
FST inversion

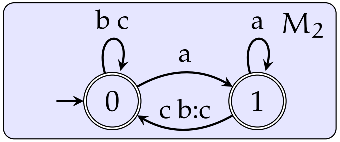
- Since an FST encodes a relation, it can be inverted
- Inverse of an FST swaps the input symbols with output symbols
- We indicate inverse of an FST M with M^{-1}





FST composition
sequential application





$aa \xrightarrow{M_1}$

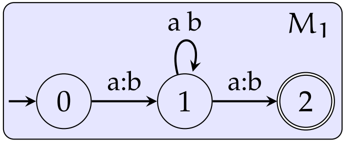
$bb \xrightarrow{M_1}$

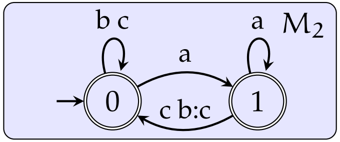
$aaaa \xrightarrow{M_1}$

$abaa \xrightarrow{M_1}$

FST composition

sequential application

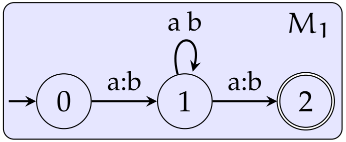


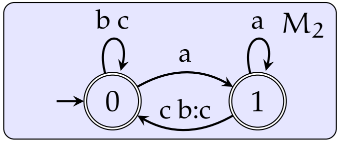


$$
\begin{array}{llll}
aa & \xrightarrow{M_1} & bb & \xrightarrow{M_2} \\
bb & \xrightarrow{M_1} & \varnothing & \xrightarrow{M_2} \\
aaaa & \xrightarrow{M_1} & baab & \xrightarrow{M_2} \\
abaa & \xrightarrow{M_1} & bbab & \xrightarrow{M_2}
\end{array}
$$

FST composition

sequential application

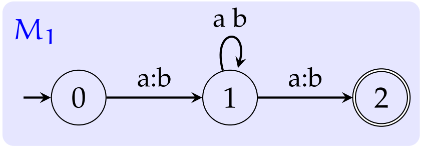


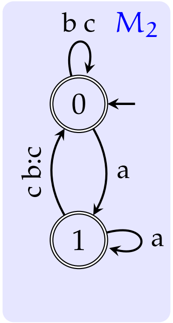


$$\begin{array}{lllll}
 & \multicolumn{3}{c}{\xrightarrow{\qquad\qquad M_2 \circ M_1 \qquad\qquad}} & \\
aa & \xrightarrow{M_1} & bb & \xrightarrow{M_2} & bb \\
bb & \xrightarrow{M_1} & \varnothing & \xrightarrow{M_2} & \varnothing \\
aaaa & \xrightarrow{M_1} & baab & \xrightarrow{M_2} & baac \\
abaa & \xrightarrow{M_1} & bbab & \xrightarrow{M_2} & bbac
\end{array}$$

- Can we compose two FSTs without running them sequentially?

FST composition





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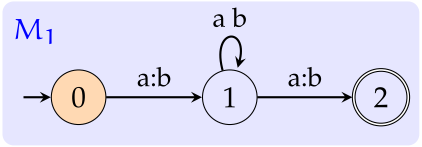
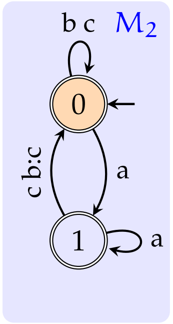
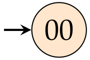
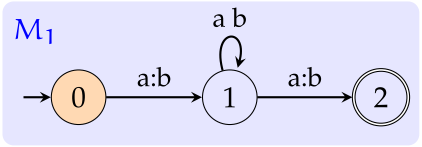
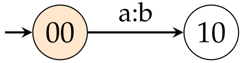


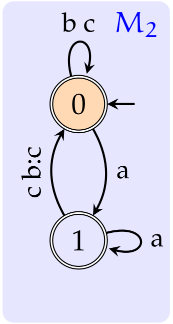




FST composition







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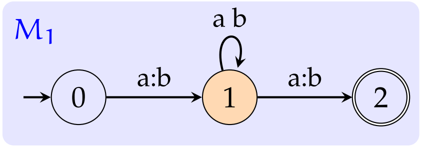

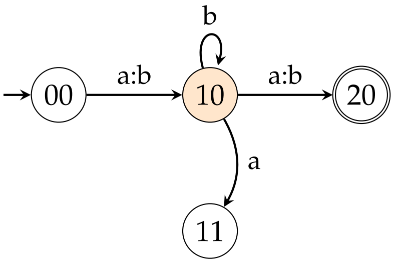

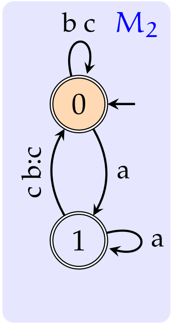

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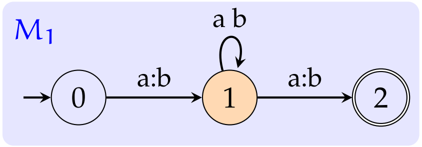

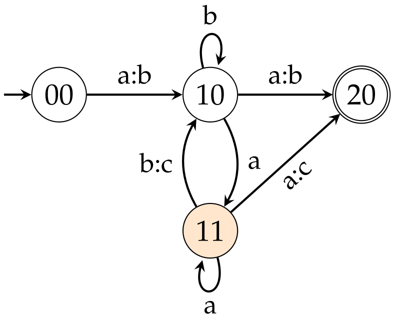

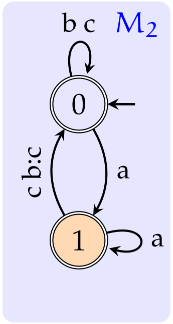

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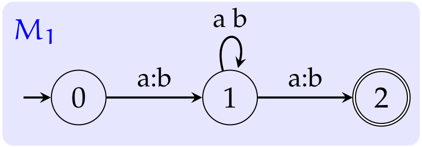
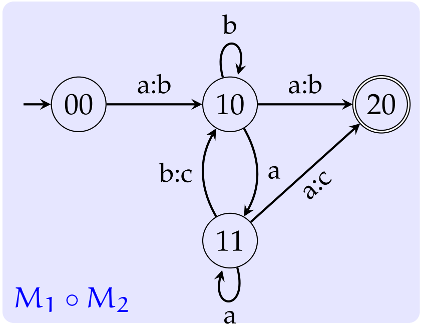
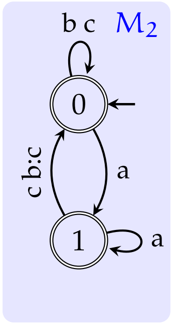
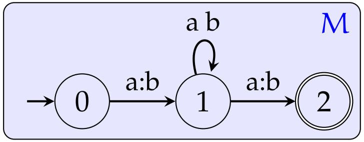


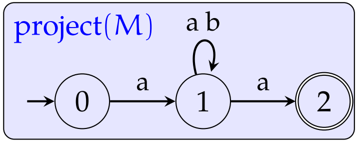




Projection

- *Projection* turns an FST into a FSA, accepting either the input language or the output language





FST determinization

- A deterministic FST has unambiguous transitions from every state on any *input* symbol
- We can extend the subset construction to FSTs
- Determinization of FSTs means converting to a *subsequential* FST
- However, not all FSTs can be determinized

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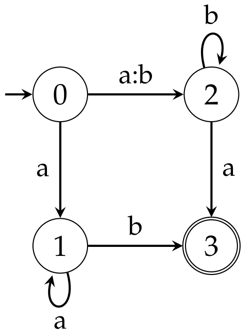
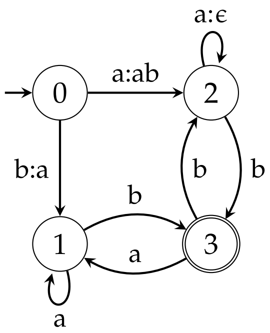
Is this FST deterministic?



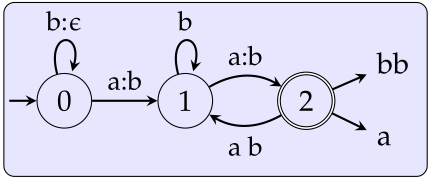
Sequential FSTs

- A sequential FST has a single transition from each state on every *input* symbol
- Output symbols can be strings, as well as ϵ
- The recognition is linear in the length of input
- However, sequential FSTs do not allow ambiguity



Subsequential FSTs

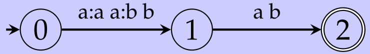
- A *k-subsequential* FST is a sequential FST which can output up to k strings at an accepting state
- Subsequential transducers allow limited ambiguity
- Recognition time is still linear



- The 2-subsequential FST above maps every string it accepts to two strings, e.g.,
  - $\mathtt{baa} \rightarrow \mathtt{bba}$
  - $\mathtt{baa} \rightarrow \mathtt{bbbb}$

An exercise

Convert the following FST to a subsequential FST



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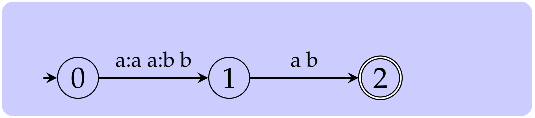
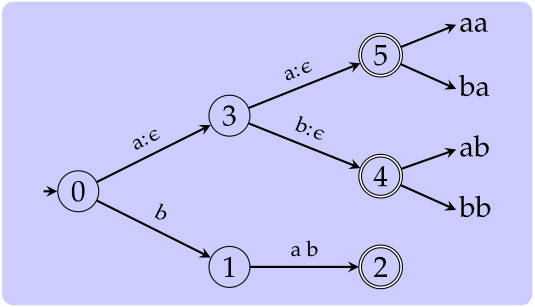
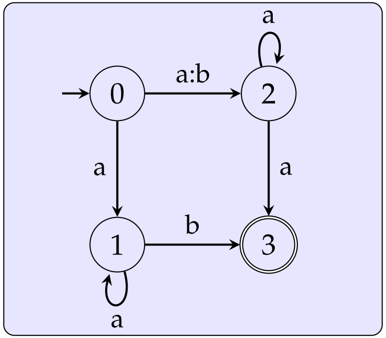




Determinizing FSTs

Another example

Can you convert the following FST to a subsequential FST?



Determinizing FSTs

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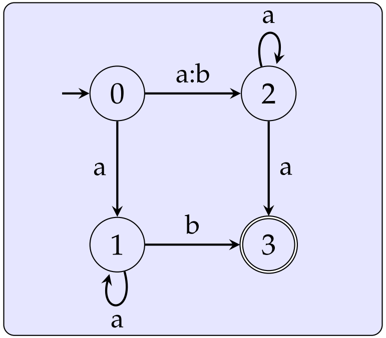
$$\begin{tikzcd}
\arrow[r] & 0 \arrow[r, "a:b"] \arrow[d, "a"'] & 2 \arrow[d, "a"] \arrow[loop above, "a"] \\
& 1 \arrow[r, "b"] \arrow[loop below, "a"] & \boxed{3}
\end{tikzcd}$$

Note that we cannot ‘determine’ the output on first input until reaching the final input.

FSA vs FST

- FSA are *acceptors*, FSTs are *transducers*
- FSA accept or reject their input, FSTs produce output(s) for the inputs they accept
- FSA define sets, FSTs define relations between sets
- FSTs share many properties of FSAs. However,
  - FSTs are not closed under intersection and complement
  - We can compose (and invert) the FSTs
  - Determinizing FSTs is not always possible
- Both FSA and FSTs can be weighted (not covered in this course)

Next:

- Parsing

References / additional reading material

- Jurafsky and Martin (2009, Ch. 3)
- Additional references include:
  - Roche and Schabes (1996) and Roche and Schabes (1997): FSTs and their use in NLP
  - Mohri (2009): weighted FSTs

References / additional reading material (cont.)

- Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second edition. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.
- Mohri, Mehryar (2009). "Weighted automata algorithms". In: *Handbook of Weighted Automata*. Monographs in Theoretical Computer Science. Springer, pp. 213–254.
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