

Finite state transducers

Data Structures and Algorithms for Computational Linguistics III
(ISCL-BA-07)

Çağrı Çöltekin
ccolt@infsa.uni-tuebingen.de

University of Tübingen
Seminar für Sprachwissenschaft

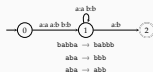
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Finite state transducers

A quick introduction

- A *finite state transducer* (FST) is a finite state machine where transitions are conditioned on pairs of symbols
- The machine moves between the states based on an *input* symbol, while it outputs the corresponding *output* symbol
- An FST encodes a *relation*, a mapping from a set to another
- The relation defined by an FST is called a *regular* (or *rational*) relation



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Introduction Operations on FSTs Determinizing FSTs Summary

Formal definition

A finite state transducer is a tuple $(\Sigma_i, \Sigma_o, Q, q_0, F, \Delta)$

Σ_i is the *input* alphabet

Σ_o is the *output* alphabet

Q a finite set of states

q_0 is the start state, $q_0 \in Q$

F is the set of accepting states, $F \subseteq Q$

Δ is a relation $(\Delta : Q \times \Sigma_i \Rightarrow Q \times \Sigma_o)$

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Where do we use FSTs?

example 1: morphological analysis



In this lecture, we treat an FSA as a simple PST that outputs its input: the edge label 'a' is a shorthand for 'aa'.

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Where do we use FSTs?

Uses in NLP/CL

- Morphological analysis
- Spelling correction
- Transliteration
- Speech recognition
- Grapheme-to-phoneme mapping
- Normalization
- Tokenization
- POS tagging (not typical, but done)
- partial parsing / chunking
- ...

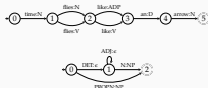
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Where do we use FSTs?

example 2: POS tagging / shallow parsing



Note: (1) It is important to express the ambiguity. (2) This gets interesting if we can 'compose' these automata.

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Closure properties of FSTs

Like FSA, FSTs are closed under some operations.

- Concatenation
- Kleene star
- Complement**
- Reversal
- Union
- Intersection**
- Inversion**
- Composition**

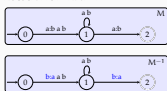
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FST inversion

- Since an FST encodes a relation, it can be inverted
- Inverse of an FST swaps the input symbols with output symbols
- We indicate inverse of an FST M with M^{-1}



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FST composition

sequential application



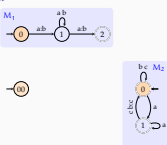
- Can we compose two FSTs without running them sequentially?

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FST composition

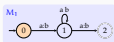


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FST composition

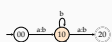
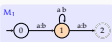


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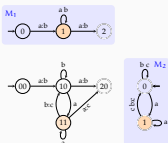
FST composition



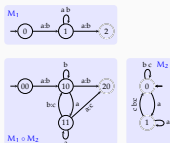
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FST composition



FST composition



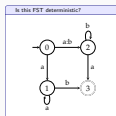
Projection

- Projection turns an FST into a FSA, accepting either the input language or the output language



FST determinization

- A deterministic FST has unambiguous transitions from every state on any input symbol
- We can extend the subset construction to FSTs
- Determinization of FSTs means converting to a *subsequential* FST
- However, not all FSTs can be determinized



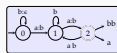
Sequential FSTs

- A sequential FST has a single transition from each state on every input symbol
- Output symbols can be strings, as well as ϵ
- The recognition is linear in the length of input
- However, sequential FSTs do not allow ambiguity



Subsequential FSTs

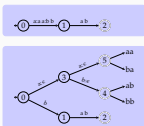
- A *k*-subsequential FST is a sequential FST which can output up to k strings at an accepting state
- Subsequential transducers allow limited ambiguity
- Recognition time is still linear



- The 2-subsequential FST above maps every string it accepts to two strings, e.g.,
 - $baa \rightarrow bba$
 - $baa \rightarrow bbbb$

An exercise

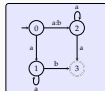
Convert the following FST to a subsequential FST



Determinizing FSTs

Another example

Can you convert the following FST to a subsequential FST?



Note that we cannot 'determine' the output on first input until reaching the final input.

FSA vs FST

- FSA are *acceptors*, FSTs are *transducers*
- FSA accept or reject their input, FSTs produce output(s) for the inputs they accept
- FSA define sets, FSTs define relations between sets
- FSTs share many properties of FSAs. However,
 - FSTs are not closed under intersection and complement
 - We can compose (and invert) the FSTs
 - Determinizing FSTs is not always possible
- Both FSA and FSTs can be *weighted* (not covered in this course)

Next:

- Parsing

References / additional reading material

- Jurafsky and Martin (2009, Ch. 3)
- Additional references include:
 - Roche and Schabes (1996) and Roche and Schabes (1997): FSTs and their use in NLP
 - Mohri (2009): weighted FSTs

References / additional reading material (cont.)

- Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*, second edition. Pearson Prentice Hall. [sasc: 978-0-13-504196-3](https://doi.org/10.1300/J3041963).
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