Top-down Chart Parsing: the Earley algorithm

Data Structures and Algorithms for Computational Linguistics III (ISCL-BA-07)

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Parsing so far

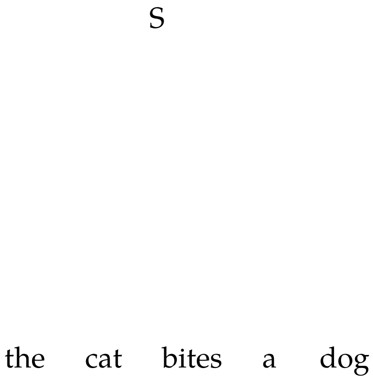
- We can formulate parsing as
  - Top-down: begin with the start symbol, try to *produce* the input string to be parsed
  - Bottom up: begin with the input, and try to *reduce* it to the start symbol
- Another aspect of a parser is its directionality. Two choices are:
  - Directional: parses processes the input left to right (right to left is also possible, but rarely used)
  - Non-directional: order is not important, typically require all input to be in memory before processing

Top-down parsing as search

the cat bites a dog

S	→	NP VP
NP	→	Det N
VP	→	V NP
VP	→	V
Det	→	a
Det	→	the
N	→	cat
N	→	dog
V	→	bites

Top-down parsing as search



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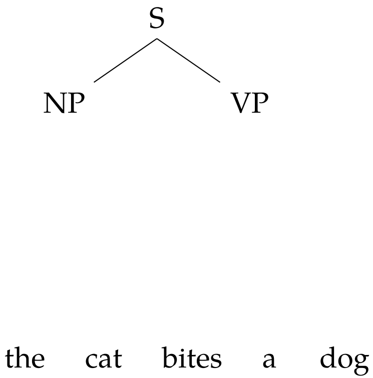

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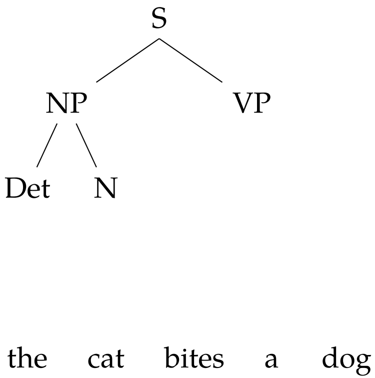

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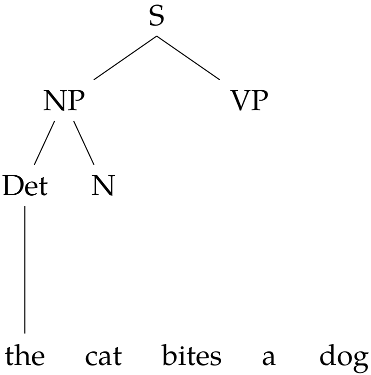

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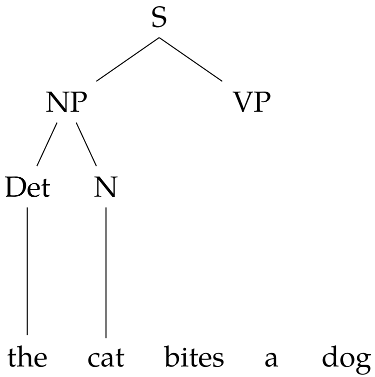

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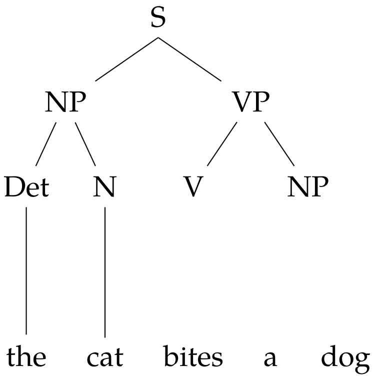

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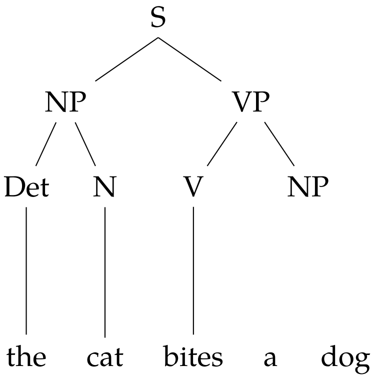

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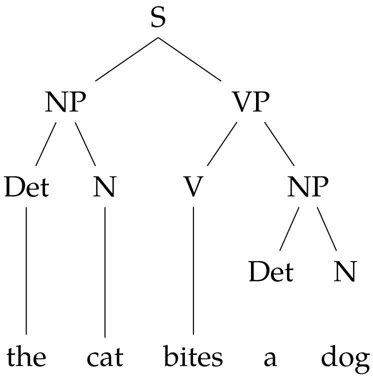

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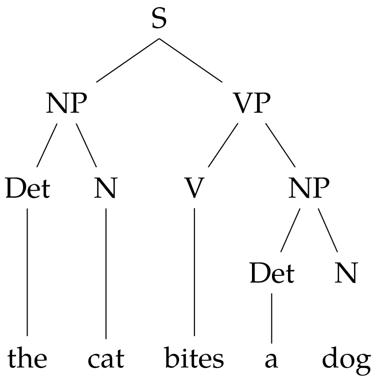

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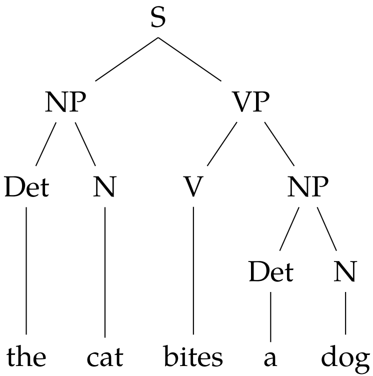

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Earley algorithm

- Earley algorithm is a top down (and left-to-right) parsing algorithm
- It allows arbitrary CFGs
- Keeps record of constituents that are

  predicted using the grammar (top-down)
  in-progress with partial evidence
  completed based on input seen so far

  at every position in the input string
- Time complexity is $O(n^3)$

Earley chart entries (states or items)

Earley chart entries are CF rules with a ‘dot’ on the RHS representing the state of the rule

- $A \rightarrow \bullet\alpha[i, i]$ predicted without any evidence (yet)
- $A \rightarrow \alpha \bullet \beta[i, j]$ partially matched
- $A \rightarrow \alpha\beta \bullet [i, j]$ completed, the non-terminal A is found in the given span

Earley algorithm: an informal sketch

1. Start at position 0, predict S
2. Predict all possible states (rules that apply)
3. Read a word
4. Update the table, advance the dot if possible
5. Go to step 2
6. If we have a completed S production at the end of the input, the input it recognized

Earley algorithm: three operations

Predictor adds all rules that are possible at the given state

Completer adds states from the earlier chart entries that match the completed state to the chart entry being processed, and advances their dot

Scanner adds a completed state to the next chart entry if the current category is a pre-terminal symbol, and the terminal symbol (word) matches

Earley parsing example (chart[0])

0 she 1 saw 2 a 3 duck 4

state	rule	position	operation
0	$\gamma \rightarrow \bullet S$	[0,0]	initialization
1	$S \rightarrow \bullet NP\ VP$	[0,0]	predictor
2	$S \rightarrow \bullet Aux\ NP\ VP$	[0,0]	predictor
3	$NP \rightarrow \bullet Det\ N$	[0,0]	predictor
4	$NP \rightarrow \bullet NP\ PP$	[0,0]	predictor
5	$NP \rightarrow \bullet Prn$	[0,0]	predictor

Note: the chart[0] is independent of the input.

S	→	NP VP
S	→	Aux NP VP
NP	→	Det N
NP	→	Prn
NP	→	NP PP
VP	→	V NP
VP	→	V
VP	→	VP PP
PP	→	Prp NP
N	→	duck
N	→	park
V	→	duck
V	→	ducks
V	→	saw
Prn	→	she | her
Prp	→	in | with
Det	→	a | the
Aux	→	does | has

Earley parsing example (chart[1])

0 she 1 saw 2 a 3 duck 4

state	rule	position	operation
6	Prn → she •	[0,1]	scanner
7	NP → Prn •	[0,1]	completer
8	S → NP •VP	[0,1]	completer
9	NP → NP •PP	[0,1]	completer
10	VP → •V NP	[1,1]	predictor
11	VP → •V	[1,1]	predictor
12	VP → •VP PP	[1,1]	predictor
13	PP → •Prp NP	[1,1]	predictor

S	→	NP VP
S	→	Aux NP VP
NP	→	Det N
NP	→	Prn
NP	→	NP PP
VP	→	V NP
VP	→	V
VP	→	VP PP
PP	→	Prp NP
N	→	duck
N	→	park
V	→	duck
V	→	ducks
V	→	saw
Prn	→	she | her
Prp	→	in | with
Det	→	a | the
Aux	→	does | has

Earley parsing example (chart[2])

0 she 1 saw 2 a 3 duck 4

state	rule	position	operation
14	V → saw •	[1,2]	scanner
15	VP → V •NP	[1,2]	completer
16	VP → V •	[1,2]	completer
17	S → NP VP •	[0,2]	completer
18	NP → •Det N	[2,2]	predictor
19	NP → •NP PP	[2,2]	predictor
20	NP → •Prn	[2,2]	predictor

S	→ NP VP
S	→ Aux NP VP
NP	→ Det N
NP	→ Prn
NP	→ NP PP
VP	→ V NP
VP	→ V
VP	→ VP PP
PP	→ Prp NP
N	→ duck
N	→ park
V	→ duck
V	→ ducks
V	→ saw
Prn	→ she | her
Prp	→ in | with
Det	→ a | the
Aux	→ does | has

Earley parsing example (chart[3])

0 she 1 saw 2 a 3 duck 4

state	rule	position	operation
21	Det → a •	[2,3]	scanner
22	NP → Det •N	[2,3]	completer

S	→	NP VP
S	→	Aux NP VP
NP	→	Det N
NP	→	Prn
NP	→	NP PP
VP	→	V NP
VP	→	V
VP	→	VP PP
PP	→	Prp NP
N	→	duck
N	→	park
V	→	duck
V	→	ducks
V	→	saw
Prn	→	she | her
Prp	→	in | with
Det	→	a | the
Aux	→	does | has

Earley parsing example (chart[4])

$_0$ she $_1$ saw $_2$ a $_3$ duck $_4$

state	rule	position	operation
23	N → duck •	[3,4]	scanner
24	V → duck •	[3,4]	scanner
25	NP → Det N •	[2,4]	completer
26	VP → V NP •	[1,4]	completer
27	S → NP VP •	[0,4]	completer

S → NP VP
S → Aux NP VP
NP → Det N
NP → Prn
NP → NP PP
VP → V NP
VP → V
VP → VP PP
PP → Prp NP
N → duck
N → park
V → duck
V → ducks
V → saw
Prn → she | her
Prp → in | with
Det → a | the
Aux → does | has

Earley parsing: summary

- Complexity (asymptotic) is the same as CKY
  - time complexity : $O(n^3)$
  - space complexity: $O(n^2)$
- Our example shows recognition, we need to maintain back links for parsing
- Again, the Earley chart stores a parse forest compactly, but extracting all trees may require exponential time

Summary

- The Earley parser is a top-down parser with bottom-up filtering (or, you can also view it the other way around)
- The parser improves over a backtracking parser by
  - dynamic programming: not re-computing the subtrees
  - filtering: not generating hypotheses (predictor) that cannot match at a given input position
- It can process any CFG (no need for CNF)
- There is a nice relation between CKY and Earley: you can view Earley as binarizing the grammar (converting to CNF) ‘on the fly’
- Suggested reading: Grune and Jacobs (2007, section 7.2)

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Next:

- Dependency parsing
- Reading suggestion: Jurafsky and Martin (2009, draft chapter 19)

An exercise

Construct the CKY and Earley charts for the sentence below

`The duck she saw is in the park`

Recommended grammar:

S → NP VP
NP → Det N
NP → Prn
NP → NP PP
NP → NP S
VP → V NP
VP → V
VP → VP PP

PP → Prp NP
N → park
N → duck
V → is
V → saw
Prn → she
Prp → in
Det → the

Acknowledgments, references, additional reading material

Grune, Dick and Ceriel J.H. Jacobs (2007). *Parsing Techniques: A Practical Guide*. second. Monographs in Computer Science. The first edition is available at `http://dickgrune.com/Books/PTAPG_1st_Edition/BookBody.pdf`. Springer New York. ISBN: 9780387689548.

Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second edition. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.